$$(a \times f) + (c \times b) + (e \times d) - (e \times b) + (a \times d) + (c \times f)$$

$$= [(af) + (b + e)] - [cb + ad + cf]$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$$

$$x_i \geq 0$$

$$b_i \geq 0$$

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Maximize $P = P_1X_1 + P_2X_2 + \dots + P_kX_k$

Subject to: $a_1X_1 + a_{12}X_2 + \dots + a_kX_k \leq q_1$

$q_{21}X_1 + a_{22}X_2 + \dots + a_{2k}X_k \leq q_2$

$\vdots$

$a_{n1}X_1 + a_{n2}X_2 + \dots + q_{nk}X_k \leq q_n$

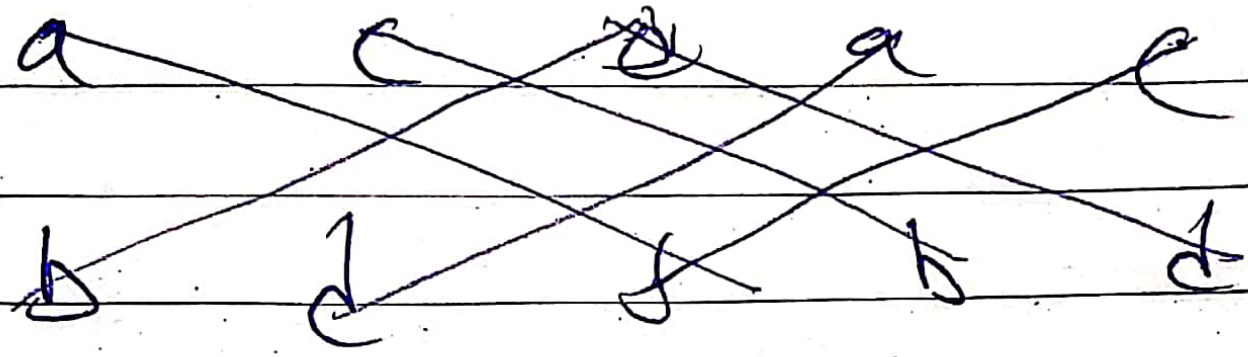
$X_1, X_2, \dots X_k \geq 0$

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Let $z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1$

$a_{21}x_1 + a_{22} + \dots + a_{2n}x_n \leq b_2$



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Assignment Problem 2

4)
$$\begin{bmatrix} (5,4) & (3,6) \\ (6,3) & (1,1) \end{bmatrix}$$

Let the set of all strategies for ith be S_i

$$S \subset S_1 \times \dots S_n$$

$$U_i(s) = \sum_{a=A} U_i(a) \Pr(a|s)$$

$$\Pr(a|s) = \prod_{j \in N} S_j(a_j)$$

Let player 2 play B with P, F 1-P

If player 1 best responds with a mixed strategy.

	B	F
B	2, 1	0, 0
F	0, 0	1, 2

$$U_1(B) = U_1(F)$$

$$2P + 0(1-P) = 0P + 1(1-P)$$

$$P = \frac{1}{3}$$

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$$\frac{p_{11}}{p_{11}+p_{12}} a_{11} + \frac{p_{12}}{p_{11}+p_{12}} a_{12} \geq \frac{p_{11}}{p_{11}+p_{12}} a_{21} + \frac{p_{12}}{p_{11}+p_{12}} a_{22}$$

$$p_{11} a_{11} + p_{12} a_{12} \geq p_{11} a_{21} + p_{12} a_{22}$$

$$p_{21} a_{21} + p_{22} a_{22} \geq p_{21} a_{11} + p_{22} a_{12}$$

$$p_{11} a_{11} + p_{21} + b_{21} \geq p_{11} b_{12} + p_{21} b_{22}$$

$$\Rightarrow \sum_{j=1}^{m} a_{ij} p_{ij} \geq \sum_{j=1}^{m} a_{qj} p_{ij} \quad \text{for all Row } i, q$$

$$\Rightarrow \sum_{i=1}^{n} b_{ij} p_{ij} \geq \sum_{i=1}^{n} b_{ir} p_{ij} \quad \text{for all col. } j, s.$$

* We always have correlated Eqm.

* Social Welfare Payoff = Sum of all individual payoffs

in pure st: (i^*, j^*) : $a_{i^*j^*} + b_{i^*j^*}$

in mixed st: (X, Y):

$$\sum_{i=1}^{n} \sum_{j=1}^{m} (a_{ij} + b_{ij}) x_i y_j = E_I(X, Y) + E_{II}(X, Y)$$

$$\max X(A+B) Y^T$$

of dist. P:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} (a_{ij} + b_{ij}) p_{ij} = E_P(I) + E_P(II)$$

Example1: $A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

(a) Pure nash eqm: Row 1, column 1

Payoffs: $P_1 = 2$, $P_2 = 1$

Social welfare = 3

$$X^* = BR_I(y) = \begin{cases} \emptyset & y > \frac{2}{5} \\ 0 & y < \frac{2}{5} \\ [0,1] & y = \frac{2}{5} \end{cases} \qquad y^* = BR_{II}(x) = \begin{cases} 1 & x > \frac{3}{5} \\ 0 & x < \frac{3}{5} \\ [0,1] & x = \frac{3}{5} \end{cases}$$

$X^* = \left(\frac{3}{5}, \frac{2}{5}\right)$ $\quad y^* = \left(\frac{2}{5}, \frac{3}{5}\right)$

$E^* = X^* A Y^{*T} = 1/5$ $\quad E_2 = X^* B Y^{*T} = 1/5$ $\quad$ Social welfare $= 2/5$

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3.4 Nonlinear Programs:

Example 1

$$\begin{bmatrix} (-1,1) & (0,2) & (0,2) \\ (2,1) & (1,-1) & (0,0) \\ (0,0) & (1,1) & (1,2) \end{bmatrix}$$

$X = (x_1, x_2, x_3)$ $\quad Y = (y_1, y_2, y_3)$

Max $XAY^T + XBY^T - p - q$ over $x_1, x_2, x_3, y_1, y_2, y_3, p, q$

Subject to

$$-y_1 \le p$$
$$2y_1 + y_2 \le p$$
$$y_2 + y_3 \le p$$

$$x_1 + x_2 \le q$$
$$2x_1 - x_2 + x_3 \le q$$
$$2x_1 + 2x_3 \le q$$

$$x_1 + x_2 + x_3 = y_1 + y_2 + y_3 = 1$$

Solution: $X = (0, 2/3, 1/3)$ $\quad Y = (1/3, 0, 2/3)$

$p = E_1(X, Y) = \frac{2}{3}$ $\quad q = E_2(X, Y) = \frac{2}{3}$

Pure nash Eqm: at (2, 1)

$X = (0, 1, 0)$ $\quad Y = (1, 0, 0)$

Expected Payoff: $E_1(X, Y) = 2$ $\quad E_2(X, Y) = 1$

at (3, 3)

$X = (0, 0, 1)$ $\quad Y = (0, 0, 1)$

$E_1(X, Y) = 1$ $\quad E_2(X, Y) = 2$

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